

## Combinations and Permutations – Grade Eight

### Ohio Standards Connection:

#### **Data Analysis and Probability**

##### Benchmark H

Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.

##### Indicator 10

Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.

#### **Mathematical Processes Benchmark**

- A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
- E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

### Lesson Summary:

*Students test various hypothetical situations to gain an understanding of the difference between a combination and a permutation. Students learn to create lists and tree diagrams to assist them in organizing information and use counting techniques to determine numerical solutions for problem situations involving combinations and permutations. The lesson embeds differentiated instructional methods for representation and communication as well as multiple opportunities to assess.*

**Estimated Duration:** Three hours

### Commentary:

Experiments, simulations and visual representations are effective methods for developing the understanding of combinations and permutations. These methods help students reveal the similarities and differences of the two concepts, which can easily be confused by eighth-grade students. Real-world applications of combinations and permutations allow students to make connections between classroom experiences and their daily lives. These counting techniques generate possible outcomes and determine the probability of a specific event.

### Pre-Assessment:

#### **Part One**

- Pair students or have students choose partners.
- Pose the following task to the students:  
*Your family is ordering a family-sized submarine sandwich. There are four toppings from which to choose (ham, cheese, lettuce, tomato). You have a coupon for a three-ingredient sandwich. Determine all the different three-ingredient sandwiches you could order.*
- Direct students to create a list, diagram or table on chart paper to show possible outcomes and counting techniques.

### **Instructional Tips:**

Consider manipulatives by providing four construction paper squares to represent ham, cheese, lettuce and tomato. Students use the construction paper squares to manipulate the ingredients. As another option, use four students to act out the selection of three toppings and have the others list and count the possible combinations of students.

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- Move around the classroom as students work. Be sure that each pair determines there are four possible combinations.
- Select students to share their solutions and strategies. Choose students so a variety of strategies may be presented. Discuss the results and methods used to complete the task through oral discussion and assess diagrams created on the chart paper. Build on student examples to show use of lists or tree diagrams. Use these questions to guide a class discussion:
  - a. How many different ways can you arrange the three items?
  - b. How do we know that we have all of the arrangements? (Ask students to share their strategies for counting.)
  - c. Does the order in which the ingredients are selected make a difference?
  - d. Does it change the sandwich? (The order does not matter; you still have the same sandwich. It was not specified that ingredients had to be in a particular order.)

### **Scoring Guidelines:**

Observe how students organize their thinking about counting outcomes. Note students who use lists, tree diagrams or invented strategies to organize the possible outcomes to be counted. Use a checklist to assess the level of understanding for individual students. A checklist can be created from a table in a word processing program like the one below.

Name	Correct # of Combinations	Method for counting (tree diagram, list, table, invented)

### *Answer Key:*

1. ham, cheese, tomato
2. ham, cheese, lettuce
3. ham, tomato, lettuce
4. cheese, tomato, lettuce

### **Part Two**

Use this pre-assessment activity with Part Two of this lesson.

- Pair students or have students choose partners:
- Pose the following task to the students:
 

*Think about the sandwich ingredient combinations you found in the previous lesson. You chose three ingredients from four. Determine how many ways you can assemble a sandwich with ONLY three ingredients (ham, cheese, tomato). This will depend on the order that ingredients are placed on the sandwich. For example, putting on ham, then tomato, then cheese is different than putting on tomato, then cheese, then ham.*
- Direct students to create a list, diagram or table on chart paper to show possible outcomes and counting techniques.

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### Instructional Tips:

Consider manipulatives by providing four construction paper squares of different colors to represent ham, cheese, lettuce and tomato. Students use the construction paper squares to manipulate the ingredients. As another option, use four students to act out the possible combinations of selecting three students in a sequential order.

- Observe partner discussions. Ask questions to check for understanding and to find out how many ways the students found. If students struggle, facilitate a class discussion to solve the problem. Select students to act as the four toppings to rearrange and others to keep track and count. Use a list during the live demonstration.
- Select students to share solutions and strategies. Choose students so a variety of strategies may be presented. Discuss the results and methods used to complete the task through oral discussion and assessing diagrams created on the chart paper. Build on student examples to show use of lists or tree diagrams. Use the questions below to guide a class discussion:
  - a. How many different ways can you arrange the three items?
  - b. How do we know that we have all of the arrangements? (Ask students to share their strategies for counting.)
  - c. Does the order in which the ingredients are selected make a difference?
  - d. Does it change the sandwich? (Yes, the order does matter.)

### Scoring Guidelines:

Through observation determine which strategies students used for organizing the items to be counted manipulatives, lists or tree diagrams. Use a checklist to assess the level of understanding and method for individual students. A checklist can be created from a table in a word processing program .

If all students demonstrate understanding of combinations in the pre-assessment activity, skip Part One of this lesson and continue to Part Two, Permutations. If students do not develop tree diagrams or create lists to show counting techniques, but are still able to figure the number of outcomes, use Part One of this lesson to develop understanding of tree diagrams and lists to create and count possible outcomes.

Name	Correct # of Permutations	Use of tree diagram or list

### Answer Key

1. ham, cheese, tomato
2. ham, tomato, cheese
3. cheese, ham, tomato
4. cheese, tomato, ham
5. tomato, cheese, ham
6. tomato, ham , cheese

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### **Post-Assessment:**

#### **Option One**

- Distribute *Combinations and Permutations Post-Assessment*, Attachment A, to each student. Students find solutions for four problem situations using either combination or permutation strategies. Students should use lists or tree diagrams to organize the information and counting techniques or formulas to provide numerical solutions for each situation.

### **Scoring Guidelines:**

Use the answer key and scoring rubric on *Combinations and Permutations Post-Assessment Option One Scoring Guideline and Answer Key*, Attachment B.

#### **Option Two**

- Assign a writing assignment to have students show understanding of combinations and permutations by
  - a. describing the similarities and differences of combinations and permutations and
  - b. creating original examples of a combination and a permutation.
- Provide a graphic organizer to organize a comparison of the similarities and differences.

### **Scoring Guidelines:**

Use *Scoring Guidelines for Post-Assessment option Two*, Attachment C.

### **Instructional Procedures:**

#### **Part One: Combinations**

1. Implement Part One of the pre-assessment activity.
2. Provide problem situations that allow students to explore combinations in a variety of applications. Divide students into groups of three or four. Assign each situation to two groups so the groups may compare results after completing the task. Sample scenarios are:
  - How many different ways can you choose two ice cream toppings from three? (Three combinations of toppings for A, B and C. Combinations are AB, AC or BC.)
  - How many different ways can four students be seated at two desks? (Six combinations, label the students A, B, C and D. Combinations are AB, AC, AD, BC, BD or CD.)
  - How many different ways can two food items be chosen from six food items? (15 combinations. Label food items A, B, C, D, E, and F. Combinations are AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF or EF.)
  - How many ways can you choose three chores from five to do before dinner (clean your room, feed the fish, take the trash out, cook dinner, wash dishes)? (10 combinations. Label the chores A, B, C, D and E. Combinations are ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE or CDE.)
3. Direct groups to create lists or tree diagrams to organize the data and then use counting techniques to determine a numerical result for each situation. Model how to create a tree diagram or list if necessary. Use a scenario such as counting how many combinations of heads and tails you might get from flipping two coins (two heads, two tails, one head and one tail).

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4. Have groups share with another group and teach that group about their problem situation, or compare solutions with the other group completing the same scenario. Assess students understanding by observing their methods for organizing (i.e., tree diagrams, lists or other) and listening to their discussions.
5. Introduce the term “combination”. Explain that they have been finding combinations, and ask them to describe/explain the processes they used and what they believe a combination is. Be sure students understand the concept and term before proceeding. Give students an opportunity to write a definition for combination in their own words in journals or notebooks.

### Instructional Tip:

The Ohio Mathematics Academic Content Standards defines a combination as “*a selection of a group of items or events from a set without regard to the order; e.g., the number of three-piece outfits from the set of clothes in the closet.*”

6. Present the following scenario and discuss reasonableness of using tree diagrams and/or lists to count possible outcomes.  
*The (your school here) Eighth Grade Mathematics Team won the county Math-a-Thon Competition. The reward was a six topping pizza. At the local pizza parlor, 10 toppings were offered. How many combinations of six-topping pizzas could be chosen?*
7. Discuss if a tree diagram or a list is appropriate to determine the total number of ways to choose six pizza toppings from 10. Why or why not? Students begin to realize that if the numbers are too large, these methods may be too tedious to be efficient.

### Instructional Tip:

Some students may be ready for the challenge of discovering the formula at this point. The Academic Content Standards Benchmarks and Indicators for grade eight do not require students to use formulas to find possible outcomes.

8. Have pairs of students create two problem situations that can be solved using combination methods for solution. Each situation should use lists or tree diagrams and counting techniques. On separate pieces of paper, students find the numerical solutions for each situation. Remind them to consider if their solutions are reasonable.
9. Have pairs exchange problem situations with another pair and solve for numerical solutions for each other’s problem situations.
10. Have pairs verify numeric solutions and methods used to find the solution.
11. Summarize the lesson by asking students to describe a combination in their own words with a partner or by writing a *Mathematics Exit Ticket*, Attachment D. They should include a definition and an example of a combination in their description. Collect the tickets as students leave at the end of the class. Use tickets as a formative assessment by quickly reading to determine what students understand after the lesson. Use the analysis to inform future instruction.

### Part Two: Permutations

12. Implement Part Two of the pre-assessment activity.

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13. Provide problem situations that allow students to explore permutations in a variety of applications. Divide students into groups of three or four. Assign each situation to two groups. Sample scenarios include:
  - How many different ways could four students exit a room?
  - How many different ways could the letters K, A and B be ordered?
  - How many different ways could the following colors be arranged in horizontal stripes to create a flag? (red, black, yellow, orange and green)
  - How many different ways could four students be placed at two desks? (Desk A, Desk B)
14. Direct the groups to organize the data and then use counting to determine a numerical result for each situation.
15. Regroup students by taking one person from each of the scenario groups and combining them. Each student shares the solutions for the problem situation and explains the strategies used. Each student in the group presents. This instructional strategy makes every student accountable for understanding the problem situation and teaching it to others.
16. Introduce the term “permutation” to students. Explain that they have been finding permutations, ask them to describe or explain the process they used and what they believe a permutation is. Be sure that students understand the concept and term before proceeding. Give students an opportunity to write a definition for permutation in their own words in journals or notebooks.

### Instructional Tip:

The Ohio Mathematics Academic Content Standards defines a permutation as “*a possible order or arrangement of a set of events or items.*”

17. Present the following scenario to discuss reasonableness of using tree diagrams and lists to count possible outcomes:

*The softball coach was putting together a batting order for the first game of the season. There are nine players whom the coach has to order. How many possible ways could the coach order the batters?*
18. Discuss whether a tree diagram or a list is appropriate to determine the total number of ways that you could arrange nine players in a batting order. Why or why not? Students will begin to realize that if the numbers are too large, these methods may be too tedious to be efficient.

### Instructional Tip:

Some students may be ready for the challenge of discovering the formula at this point. The Mathematics Academic Content Standards Benchmarks and Indicators for grade eight do not require students to use formulas to find possible outcomes.

19. Have students create two problem situations that use permutation methods to find possible outcomes. Students solve situation using lists or tree diagrams and counting techniques. On separate pieces of paper, students find the numerical solutions for each situation. Remind them to consider if their solutions are reasonable.

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20. Instruct pairs to exchange problem situations with another pair. The pairs continue to exchange papers and solve problems until about 10 minutes remain in the period. Observe student discussions and solutions.
21. Pairs join those with whom they exchanged papers to verify numeric solutions and methods used.
22. Summarize the lesson by asking students to describe a permutation in their own words to partners or by writing a “Mathematics Exit Ticket.” Students must include a definition and an example of a combination in their description. Ask students to compare combinations and permutations. Collect the tickets and read to determine which students can distinguish combinations from permutations.

### Part 3: Combinations and Permutations

23. Engage students in thinking about times when they use combinations and permutations in real-life situations:  
*Think of a time when it matters that you use a permutation or combination in real life.*  
If students do not respond well, ask them to think about a daily activity where order matters: brushing teeth, getting dressed, baking a cake, building a house, etc. Have students think first, find partners and share their ideas. Select students to share examples.
24. Pose the following task to students:  
*A new student comes to your school. This student needs a schedule of core academic classes (mathematics, language arts, science and social studies). What are the classes this student must take? How many schedules could be created? Design multiple schedules for this student.*
25. Allow groups to collaborate to create possible schedules.
26. Ask students if they have all of the possible schedules, and how they can be sure (use an organized list or a tree diagram). Ask if they have listed permutations or combinations and how they know. (These are permutations because the order matters.)
27. Direct students to identify the combination of classes in this task. (Simply, the given set of classes.)
  - What if the social studies teacher is available only first and second periods? Would all of the schedules apply? Why or why not?
28. Pose the question:  
*Is a combination lock a combination?*  
Allow students to discuss with partners, or individually write the answer on slips of paper. Tell students to write brief explanations to support their reasoning. Collect paper and discuss responses as a class.
29. Complete post-assessment activities.

### **Differentiated Instructional Support:**

Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

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- Use cooperative-learning methods such as partner and small-group discussion and students teaching one another to make all students accountable for learning the content and to provide a pathway for diverse learners to process new information related to the concept.
- Allow students to manipulate items or act out scenarios to represent the combinations and permutations during assessment and instruction.
- Students may choose a post-assessment which may appeal to their learning preferences.
- Provide students problem situations that require a smaller number of outcomes.
- Require students with a complete understanding of the content and processes of the benchmark and indicator to explore strategies leading to the development of formulas to find the number of possible outcomes for larger numbers and sets.

### **Extensions:**

- Students find distinguishable permutations of letters for words like mime, sheet or Mississippi, which have repeated letters. Then, they can create a rule for the situation (mime, mmie, mmei, imme, iemm, eimm, emmi).
- In groups of four, students each shake hands with every other student. How many unique sets of handshakes are there if each member of the group is to shake every other member of the group's hand? Remind students to use what they have learned to keep track (i.e., list or tree diagrams). Act out the handshakes and record each handshake. Discuss the solution as a class; be sure to include the importance of uniqueness when dealing with combinations (i.e.,  $a-b = b-a$ ). Compare and contrast this situation (combination) and permutation situations. How many handshakes in groups of eight? 12? whole class?

### **Homework Options and Home Connections:**

- Assign the following situation that students can complete at home with materials.  
*How many combinations of three utensils (fork, spoon and knife) can you create if you have a soup spoon, teaspoon, salad fork (shorter prongs), dinner fork, butter knife and steak knife?"*
  - a. How can this be used to create a combination?
  - b. How can this be used to create a permutation?
  - c. What is the number of combinations?
  - d. What is the number of permutations?
  - e. Draw a tree diagram or list to support your answers.

### **Materials and Resources:**

*The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site's main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.*





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*For the teacher:* Chart paper and markers

*For the student:* Chart paper and marker, counters

### **Vocabulary:**

- combination
- permutation
- possible outcome
- tree diagram

### **Technology Connections:**

- Many graphing calculators have permutation ( $nPr$ ) and combination ( $nCr$ ) functions to calculate the number of permutations or combinations of  $n$  objects taken  $r$  at a time.

### **Research Connections:**

Marzano, Robert J., Jane E. Pollock and Debra Pickering. *Classroom Instruction that Works: Research-Based Strategies for Increasing Student Achievement*, Alexandria, Va: Association for Supervision and Curriculum Development, 2001.

National Council of Teachers of Mathematics. Principles and Standards in School Mathematics. Reston, Va: NCTM, 2000.

Sousa, David A. *How the Brain Learns: A Classroom Teacher's Guide*. Reston, Va: NASSP, 1995.

### **Attachments:**

Attachment A, *Combinations and Permutations Post-Assessment*

Attachment B, *Post-Assessment Option One, Scoring Guidelines and Answer Key*

Attachment C, *Scoring Guidelines for Post-Assessment Option Two*

Attachment D, *Mathematics Exit Ticket*



## Combinations and Permutations – Grade Eight

### Attachment A

### Combinations & Permutations Post-Assessment

Name \_\_\_\_\_

Date \_\_\_\_\_

**Directions:** For each problem situation, determine if solving the problem requires combinations or permutations, find the numerical value that represents the solution and explain why you chose the approach used. Show the work for your solution on a separate sheet of paper.

1. Elections are being held for student representatives, and next year's freshman class will determine which two students will be chosen. If five students are running for election, how many different groups of two can be elected to represent the freshman class?

Solve using: \_\_\_\_\_ combinations    \_\_\_\_\_ permutations (select one)

Explain your selection:

2. The new computer desk for your room has enough shelf space to have three reference books. Your parents purchased a set of four reference books for you. How many different ways can you place three of the reference books on the shelf of your new desk?

Solve using: \_\_\_\_\_ combinations    \_\_\_\_\_ permutations (select one)

Explain your selection:

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### Attachment A (Continued)

#### Combinations & Permutations Post-Assessment

3. Monograms are traditionally written with your last initial in the middle surrounded by your first and middle initials. For example, if your name is Chris Jordan Smith, then your monogram would be written as follows:

C<sup>S</sup>J

If you wanted to ignore tradition, how many different ways could you arrange the letters for your monogram if your name were Chris Jordan Smith?

Solve using: \_\_\_\_\_ combinations    \_\_\_\_\_ permutations (select one)

Explain your selection:

4. Your birthday is coming up and you are having a pizza party. You and your friends love to eat pizza and you all enjoy trying pizzas with different toppings. Your favorite pizza parlor only offers five toppings and your budget for the party will allow you to purchase pizzas with only three toppings. If no pizza can have one topping twice, how many three-topping pizzas do you and your friends have to choose from?

Solve using: \_\_\_\_\_ combinations    \_\_\_\_\_ permutations (select one)

Explain your selection:

## Combinations and Permutations – Grade Eight

### Attachment B

### Combinations and Permutations Post-Assessment Option One Scoring Guideline and Answer Key

#### *Rubric*

<b>4</b>	<ul style="list-style-type: none"> <li>All four problem situations are correctly labeled as combination or permutation.</li> <li>Explanation is accurate and clearly communicated.</li> <li>Tree or list is created to show counting technique.</li> </ul>
<b>3</b>	<ul style="list-style-type: none"> <li>Four correct numerical answers and two explanations and organizing techniques are shown.</li> <li>Three problem situations are correctly labeled as combination or permutation.</li> <li>Explanation is accurate and communicated.</li> <li>Tree or list is created to show counting technique.</li> </ul>
<b>2</b>	<ul style="list-style-type: none"> <li>One to two problem situations are correctly labeled as combination or permutation.</li> <li>Explanation contains minor errors and may show minor gaps in understanding.</li> <li>Tree or list is attempted to show counting technique.</li> </ul>
<b>1</b>	<ul style="list-style-type: none"> <li>Understanding of combinations and permutations is switched, all are incorrect.</li> <li>Explanation contains major errors and may show major gaps in understanding.</li> <li>Tree or list is poorly attempted or not attempted to show counting technique.</li> </ul>
<b>0</b>	<ul style="list-style-type: none"> <li>No evidence of understanding is demonstrated in identifying or counting possible outcomes.</li> <li>No attempt is made to complete the assignment.</li> </ul>

#### *Answer Key:*

- (Election, choose two people from five)  
This is a combination, because it does not matter in which order the two are chosen. The answer is 10. Label the people A, B, C, D and E. The possible combinations are AB, AC, AD, AE, BC, BD, BE, CD, CE and DE.
- (Computer desk/books, arrange three from four)  
This is a permutation, because order matters. The answer is 24. There are four possible combinations (A,B,C; A,B,D; A,C,D and B,C,D). Since can be arranged each of these in six different ways, there are 24 permutations.
- (Monogram, arrange three letters)  
This is a permutation, because order matters. The answer is six. The permutations are CSJ, CJS, JSC, JCS, SCJ, SJC.
- (Pizza toppings, choose three from five)  
This is a combination, because order does not matter. The answer is 10. The combinations are CDE, BDE, BCE, BCD, ADE, ACE, ACD, ABE, ABD, ABC.

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### Attachment C

#### Scoring Guidelines for Post-Assessment Option Two

<b>4</b>	<ul style="list-style-type: none"> <li>• Difference between combination and permutation is clearly stated.</li> <li>• One example of combination is provided and identified.</li> <li>• One example of permutation is provided and identified.</li> </ul>
<b>3</b>	<ul style="list-style-type: none"> <li>• Difference between combination and permutation is clearly stated.</li> <li>• Only one example of <b><u>either</u></b> combination <b><u>or</u></b> permutation is provided and identified.</li> </ul>
<b>2</b>	<ul style="list-style-type: none"> <li>• Difference between combination and permutation is <b><u>not</u></b> clearly stated.</li> <li>• One example of combination is provided and identified.</li> <li>• One example of permutation is provided and identified.</li> </ul>
<b>1</b>	<ul style="list-style-type: none"> <li>• Difference between combination and permutation is clearly stated.</li> <li>• No example of combination is provided.</li> <li>• No example of permutation is provided.</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>• Difference between combination and permutation is <b><u>not</u></b> clearly stated.</li> <li>• Only one example of <b><u>either</u></b> combination <b><u>or</u></b> permutation is provided and identified.</li> </ul>
<b>0</b>	<ul style="list-style-type: none"> <li>• Contains inaccurate comparisons, or characteristics of combinations and permutations are switched.</li> </ul>

## Combinations and Permutations – Grade Eight

### Attachment D Mathematics Exit Ticket

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Directions:** Complete the ticket by answering the question(s) given to summarize today's lesson.

### Mathematics Exit Ticket

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**Evidence of Understanding/Examples:**

